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# Simulation Analysis of Positioning for Probes in Chinese CE-3 Mission

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#### Abstract

The Chinese lunar exploration spacecraft Chang'E-3 satellite will be launched during 2012-2013. In this mission, the two kinds of probes, lander and rover, will arrive at the surface of the moon. It will bring new challenges to technology and theories compared with traditional VLBI in current CE missions. As the  $\Delta$ DOR and SBI methods are adopted, the old models are updated synchronously. Besides, after the lander lands it will stand still on the moon as time passes. Observations at varying times at stationary coordinates within the body-fixed system of the moon will be accumulated, an adjusted algorithm will be used, and the precision of angular position should be greatly improved. Moreover, the rover will move and stop in different sessions. So the D-VLBI models will also be different in the two situations, and the Kalman filter method will be introduced to solve the parameters. This paper reports the simulation analysis results in all the above cases in the CE-3 mission, including the verification of positioning models, discussions aiming at the possible problems that arise in practice, precision evaluation, etc.

#### 1. Introduction

As the development of advanced technology of deep space exploration, increasingly more targets were gradually selected to be observed, including the moon, as a hot topic in recent years. It was appealing to about one hundred circumvolant lunar spacecrafts launched by different countries. The Chinese Chang'E lunar exploration project was initiated in 2004. The main means adopted are to process the VLBI observations in near real time, taken in charge by the orbit determination subsystem-Shanghai Astronomical Observatory. The Chang'E lunar exploration is divided into three stages: orbiting, landing, and returning. During the orbiting stage, CE-1 and CE-2 probes were successfully launched in 2007 and 2010 respectively. They flew around the moon and took photos of possible landing areas, which would be used as a significant reference for the task of the next stage, soft-landing and lunar surface walking in the CE-3 mission. Specifically, the CE-1 experienced earth-moon transfer phase through several time orbital adjustments, entered a near circular orbit around the moon at the height of approximately 100 km, and finally made a controlled landing on the moon after 494 days of continuous observation were accumulated in 2009 [1]. Whereas, CE-2 was directly injected into the trans-lunar orbit, entering into a  $100 \times 100$  km orbit and descending to 15 km to obtain the photograph of Sinus Iridum area. The noise of time delay is at the level of 2-5 ns in the CE-1 mission. Since positioning results mainly depend on the precision of the observation due to the characteristics of the Instantaneous State Vector method [2], the traditional VLBI solution can only reach the precision of level 1. Nevertheless, it can play a pivotal role in the orbital maneuver such as lunar capture, soft-landing, surface walking, etc. The orbit elements were used to demonstrate the evolution of the orbital figure and location in real time. As great improvements, the X-band experiment was observed during the period. DBBC replaced

ABBC to overcome the non-linear phase frequency response, and more GPS and extragalactic radio sources were adopted to fit the clock drifts and calibrate the instruments' delay. In addition, bandwidth synthesis of X-band signal with 40 MHz bandwidth was used in the post-processing mode. In the post-processing mode, the delay data noise reaches 0.2 ns with an improvement of 1 order of magnitude compared with E-1 [3].  $\Delta DOR$  technology was also introduced to eliminate or abate the common errors in the radial direction. In follow-on CE-3 mission,  $\Delta DOR$  and SBI (Same Beam Interferometry) methods were taken to achieve higher precision [4]. This paper presents the relative theoretical models and corresponding results with simulation data.

# 2. Data Simulation and Strategies

• Static positioning for lander

In the follow-on CE-3 mission, the lander will stop on the moon for automatic exploration. During this period, observations will be accumulated at varying times. Under this condition, the multi-wave front and multi-station solution will be possible instead of same wave-front. We generated a uniform and smooth satellite orbit in the Geocentric Celestial Reference Frame with fixed lunar latitude and longitude of 30 and 40 degrees respectively. Time series of delay, rate, USB ranging and doppler for October 2012 were obtained hypothetically according to the orbit. EOP predictions for the whole year from EOP-PCC were adopted. The sampling interval was five seconds. The random errors added on time delay and ranging were set to different values to test the positioning ability. The main strategy for this case has to divide all the time into several sessions and then solve for the positions with different time sessions and different observation noise. Time delay and ranging were used as the observations. The observation equation and corresponding deviation are as follows:

For time delay,

$$c\tau = \rho_2 - \rho_1 = |\overrightarrow{r_0} - \overrightarrow{r_2}| - |\overrightarrow{r_0} - \overrightarrow{r_1}|$$

$$\frac{\partial c\tau}{\partial \overrightarrow{r_0}} = \frac{(\overrightarrow{r_0} - \overrightarrow{r_2})}{\rho_2} - \frac{(\overrightarrow{r_0} - \overrightarrow{r_1})}{\rho_1}$$

For ranging,

$$r = \frac{\rho_2 + \rho_1}{2}$$

$$\frac{\partial \rho}{\partial \overrightarrow{r_0}} = \frac{(\overrightarrow{r_0} - \overrightarrow{r_2}) + (\overrightarrow{r_0} - \overrightarrow{r_1})}{2}$$

 $\rho_1$  and  $\rho_2$  are the distances from the lander to two stations respectively;  $\overrightarrow{r_0}, \overrightarrow{r_1},$  and  $\overrightarrow{r_2}$  are the vectors in the geocentric frame of the lander and two receivers.  $\tau$  is the time delay, and r is the average of the distances from the lander to each station.

• Relative positioning for the lander and the rover when they stand still;

When the rover also lands on the lunar surface, delay and rate will be the observation. The differential model for both lander and rover were expressed as follows,

$$c\delta\tau = (\rho_4 - \rho_3) - (\rho_2 - \rho_1)$$

 $\rho_1$  and  $\rho_2$  are the distances from the lander to each station.  $\rho_3$  and  $\rho_4$  are the distances from the rover to each station.

The derivative with respect to the position of the rover in the GCRF was as follows,

$$\frac{\partial c \delta_{\overline{T}}}{\partial \overrightarrow{r_r'}} = \frac{(\overrightarrow{r_r'} - \overrightarrow{r_2})}{\rho_4} - \frac{(\overrightarrow{r_r'} - \overrightarrow{r_1})}{\rho_3}$$

 $\overrightarrow{r_r}$ ,  $\overrightarrow{r_l}$ ,  $\overrightarrow{r_1}$ , and  $\overrightarrow{r_2}$  are the position vectors in the GCRF of the rover, the lander, and the two

Suppose the relationship between  $\overrightarrow{r}$  and  $\overrightarrow{v}$  is linear during every short time interval.

$$\frac{\partial c \delta \tau}{\partial \overrightarrow{v_0}} = \frac{\partial c \delta \tau}{\partial \overrightarrow{r_r}} \Delta t$$

 $\overrightarrow{v_0}$  is the velocity of the rover.  $\Delta t$  is the time interval during which a linear relationship between coordinate and velocity is assured.

As for the rate, the observation equation will be as follows,

$$c\dot{\tau} = \frac{(\overrightarrow{r_2} - \overrightarrow{r_r})(\overrightarrow{v_2} - \overrightarrow{v_r})}{\rho_4} - \frac{(\overrightarrow{r_1} - \overrightarrow{r_r})(\overrightarrow{v_1} - \overrightarrow{v_r})}{\rho_3}$$

 $c\dot{\tau} = \frac{(\overrightarrow{r_2} - \overrightarrow{r_r})(\overrightarrow{v_2} - \overrightarrow{v_r})}{\rho_4} - \frac{(\overrightarrow{r_1} - \overrightarrow{r_r})(\overrightarrow{v_1} - \overrightarrow{v_r})}{\rho_3}$  $\overrightarrow{v_1}$ ,  $\overrightarrow{v_2}$ , and  $\overrightarrow{v_r}$  refer to the velocity of the two stations and the rover, respectively.

The derivative with respect to the position and the velocity of the rover in the GCRF are expressed as follows.

$$\frac{\frac{\partial c\delta\dot{\tau}}{\partial \overrightarrow{r_r}}}{\frac{\partial c\delta\dot{\tau}}{\partial \overrightarrow{v_r}}} = \frac{(\overrightarrow{v_r} - \overrightarrow{v_2})\rho_4^2 - (\overrightarrow{r_r} - \overrightarrow{r_2})^2(\overrightarrow{v_r} - \overrightarrow{v_2})}{\rho_4^3} - \frac{(\overrightarrow{r_r} - \overrightarrow{r_1})\rho_3^2 - (\overrightarrow{r_r} - \overrightarrow{r_1})^2(\overrightarrow{v_r} - \overrightarrow{v_1})}{\rho_3^3}$$

$$\frac{\partial c\delta\dot{\tau}}{\partial \overrightarrow{v_r}} = \frac{\partial c\delta\dot{\tau}}{\partial \overrightarrow{r_r}}\delta t$$

• When the rover walks on the lunar surface, the relative model will be similar to the above case, but with the Kalman filter algorithm introduced into the solution. The basic formulas

$$\begin{split} \widehat{X}_{k,k-1} &= f(\widehat{X}_{k-1}, k-1) \\ D_{k,k-1} &= \Phi_{k,k-1} D_{k-1} \Phi_{k,k-1}^T \\ K_k &= D_{k,k-1} H_k^T [H_k D_{k,k-1} H_k^T + R_k]^{-1} \\ \widehat{X}_k &= \widehat{X}_{k,k-1} + K_k [Z_k - h(\widehat{X}_{k,k-1}, k)] \\ D_k &= [I - K_k H_k] D_{k,k-1} \end{split}$$

 $\hat{X}_{k,k-1}$  and  $\hat{X}_{k-1}$  are the positions in the K and K-1 epoch. f is the relationship equation between the two vectors.  $D_{k,k-1}$  and  $D_{k-1}$  represent the corresponding covariances.  $Z_k$  is the observation, and h is the relative coefficient matrix. The above equation will be used iteratively to solve for D and X in every epoch.

## 3. Results Analysis

Table 1 shows the precision with different systematic errors in case 1. As the observation error increases, the position accuracy will decrease. In the table, B, L, and R represent the latitude, the longitude, and the selenocentric distance of the lander in the body-fixed system of the moon, respectively.

As is shown in Figure 1, with the different random errors, the precision will be better than 1 m after 10 minutes of accumulated observations except in the case in which two stations are removed.

		0 sys	0.1 ns sys	0.2 ns sys	0.5 ns sys	1 ns sys
	В	$0.013 \pm 2.553$	$55.814 \pm 2.553$	$111.165 \pm 2.553$	$277.786 \pm 2.554$	$555.470 \pm 2.554$
5	L	$0.027 \pm 5.257$	$120.918 \pm 5.258$	$240.842 \pm 5.259$	$601.996 \pm 5.261$	$1204.143 \pm 5.264$
	R	$-0.019 \pm 3.698$	$-84.685 \pm 3.697$	$-168.651 \pm 3.697$	$-421.364 \pm 3.696$	$-842.224 \pm 3.694$
	В	$0.006 \pm 1.805$	$118.845 \pm 1.805$	$237.236 \pm 1.805$	$593.020 \pm 1.805$	$1185.913 \pm 1.806$
10	L	$0.012\pm\ 3.716$	$250.878 \pm 3.717$	$500.817 \pm 3.718$	$1252.333 \pm 3.721$	$2505.710 \pm 3.726$
	R	$-0.008 \pm 2.614$	$-175.992 \pm 2.613$	$-351.244 \pm 2.613$	$-877.675 \pm 2.611$	$-1753.962 \pm 2.608$
	В	$0.012 \pm 1.274$	$135.611 \pm 1.274$	$270.868 \pm 1.275$	$676.966 \pm 1.275$	$1353.756 \pm 1.275$
20	L	$0.025 \pm 2.624$	$285.430 \pm 2.625$	$570.143 \pm 2.625$	$1425.489 \pm 2.628$	$2852.309 \pm 2.632$
	R	$-0.017 \pm 1.845$	$-200.255 \pm 1.845$	$-399.902 \pm 1.844$	-999.021± 1.843	$-1996.235 \pm 1.841$
	В	$-0.011 \pm 1.038$	$137.474 \pm 1.038$	$274.660 \pm 1.038$	$685.543 \pm 1.038$	$1370.854 \pm 1.039$
30	L	$-0.022 \pm 2.137$	$289.242 \pm\ 2.138$	$577.903 \pm 2.138$	$1443.033 \pm 2.141$	$2887.323 \pm 2.144$
	R	$0.016 \pm 1.503$	$-202.917 \pm 1.503$	$-405.320 \pm 1.502$	$-1011.240 \pm 1.501$	$-2020.556 \pm 1.499$

Table 1. Comparison with different systematic errors with range constraint (unit: m).

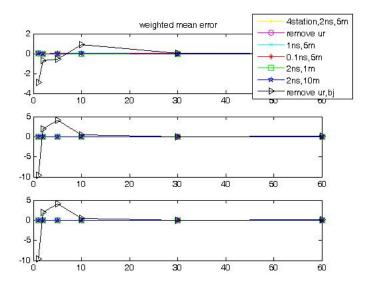


Figure 1. Positioning results comparison with different errors in the observations.

# 4. Conclusion

In the follow-on CE-3 mission, the lander and the rover will be sent onto the moon. Positioning for the two targets in different phases will be significant for the project accomplishment. This paper presented the methods to position in the three cases in the CE-3 mission and analyzed the results from each case. Adding different systematic errors and random errors will influence the positioning results. To achieve the precision of 1 m order of magnitude, the integration interval should be more than 10 minutes.

### References

- [1] Li, J., L. Guo, Z. Qian, et al., Determination of the controlled landing trajectory of Chang'E-1 satellite and the coordinate analysis of the landing point on the Moon. China SCi Bull,55(13):1240-1245, 2010.
- [2] Li, J., L. Guo, Z. Qian, et al., The application of the instantaneous states reduction to the orbital monitering of pivotal arcs of the Chang'E-1 satellie.Sci China Ser G-Phys Mech Astron, 52(12): 1833-1841, 2009.
- [3] Li, P., X. Hu, Y. Huang, et al. Orbit determination for Chang'E-2 lunar probe and evaluation of lunar gravity models. Sci China-Phys Mech 55:514-522, 2012.
- [4] Liu, Q., M. Chen, W. Xiong, et al., Relative position determination of a lunar rover by using high-accuracy multi-frequency same-beam VLBI. Sci China-Phys Mech:53, 2010.